#### JPL

### Control of Coupled Oscillator Arrays for Communications Applications

Ronald J. Pogorzelski
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California 91109

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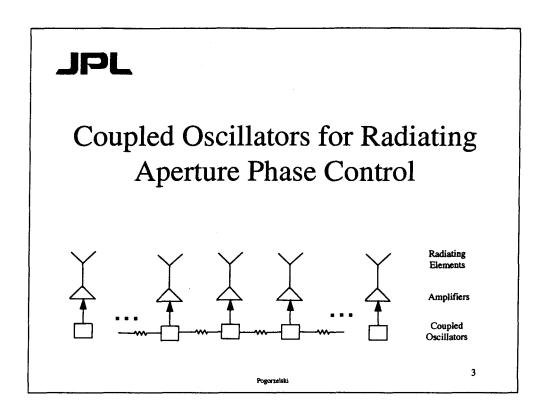
#### Introduction

- Consider a linear array of coupled oscillators.
  - Achieves high radiated power through coherent spatial power combining.
  - Usually designed to produce constant aperture phase.
- Oscillators are injection locked to each other or to a master oscillator to produce coherent radiation.
- Oscillators do not necessarily oscillate at their tuning frequency.
- Adler has shown that the phase of each oscillator is a function of the difference between the tuning frequency and the oscillation frequency.
- York, et. al. have shown that the oscillation frequency of the array is the average of the free running frequencies of the oscillators.

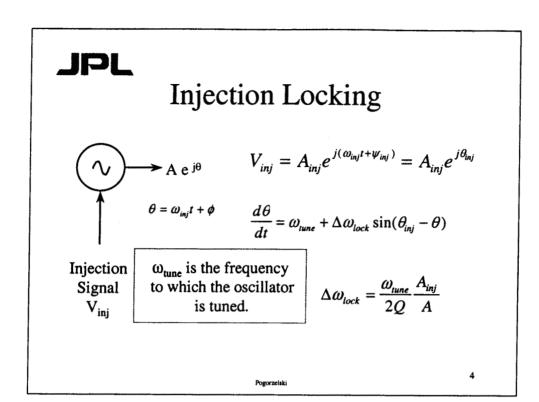
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Our purpose in coupling oscillators together is to achieve high radiated power through the spatial power combining which results when the oscillators are injection locked to each other. Adler has shown that the phase of the oscillator output is a function of the difference between the free running frequency and the oscillation frequency. York, et. al. have shown that, left to themselves, the ensemble of injection locked oscillators oscillate at the average of the tuning frequencies of all the oscillators.



This shows the concept of controlling the phase in a radiating aperture using coupled electronic oscillators. The amplifiers serve two purposes; they provide for high radiated power and they isolate the oscillators from the parasitic coupling between the radiating elements thus permitting more precise control of the nature of the interoscillator coupling.



Consider a single injection locked oscillator. We represent the signals as complex functions as indicated. In steady state, of course, the oscillator will oscillate at the injection frequency. The transient (time varying) behavior is governed by the indicated differential equation. Using this equation we can formulate the theory of a set of coupled oscillators.

# JPL Coupled Oscillators

$$\frac{d\theta_i}{dt} = \omega_{\text{none}} - \frac{\omega_i}{2Q} \sum_{\substack{j=i-1\\j\neq i}}^{j=i+1} \varepsilon_{ij} \frac{A_j}{A_i} \sin(\Phi_{ij} + \theta_i - \theta_j)$$

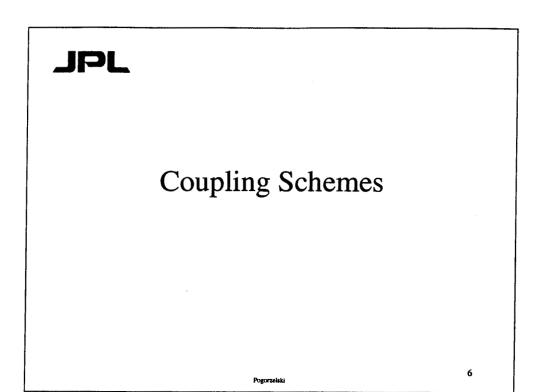
In the continuum model:

$$\nabla^2 \theta - \frac{\partial \theta}{\partial \tau} = -\frac{\omega_{tune}}{\Delta \omega_{lock}}$$

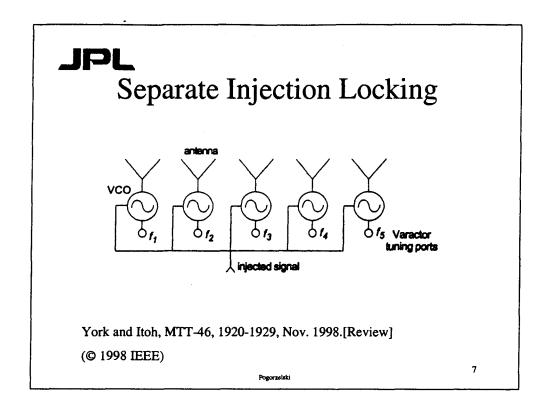
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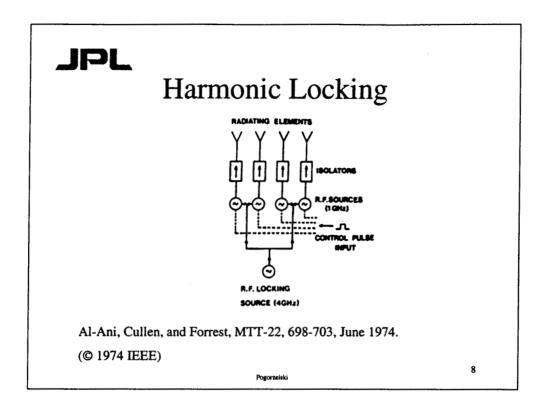
Here we adapt the preceding differential equation to describe the behavior of a linear array of coupled oscillators with nearest neighbor coupling. Using a continuum model of this description leads to the partial differential equation shown at the bottom of the vugraph. Tau is time multiplied by the locking bandwidth of the oscillators.



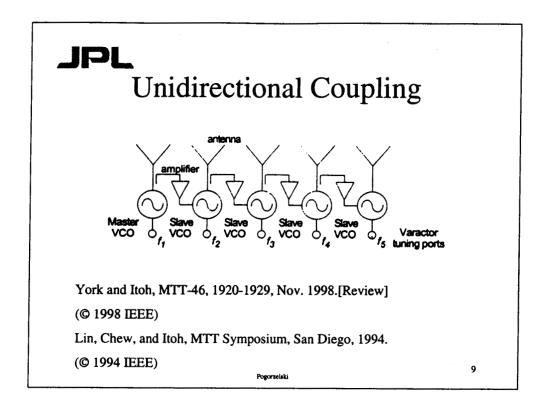
A number of coupling schemes for oscillator arrays have been proposed and/or experimentally investigated during past several years. This will be a short top level review of these and their attributes in terms of phased array control.



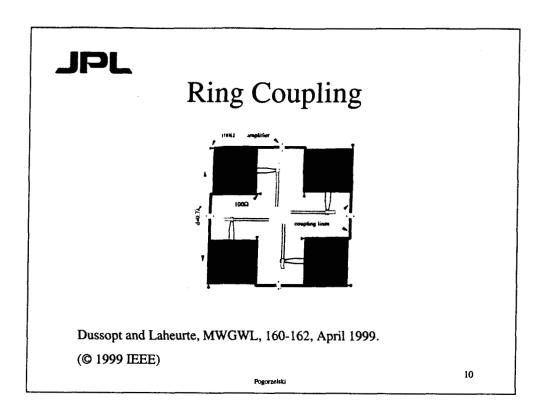
Here each oscillator is directly locked to the master oscillator and its phase relative to the master oscillator is adjustable by variation of its free running frequency.



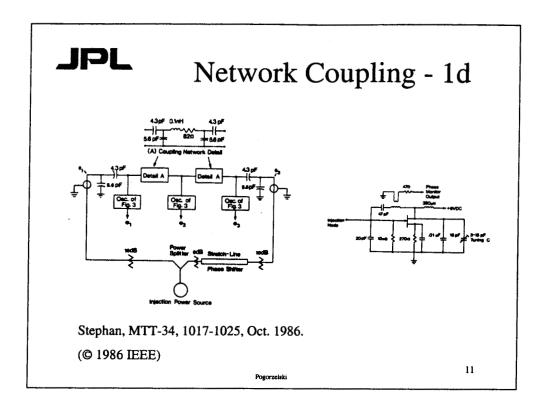
The array oscillators are locked to a master oscillator at four (n) times their frequency. Then control pulses are applied to the tuning ports which causes temporary loss of lock for a duration of one cycle of the master oscillator after which lock is reacquired. The result is a phase shift of 2 pi /4 or 90 degrees. The direction of shift is dependent on the sign of the control pulse.



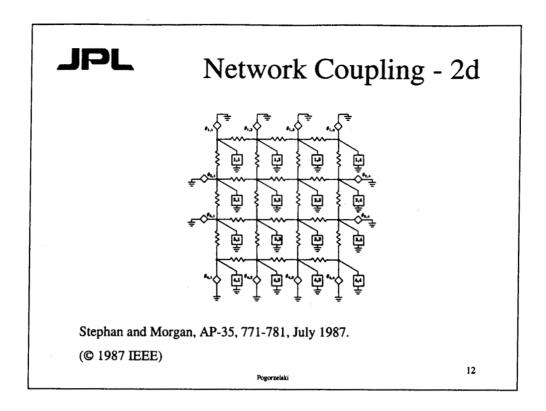
In this scheme each oscillator is injection locked to its predecessor in the chain and this implies that the relative phase of the oscillators can be adjusted by varying their free running frequencies in the manner described by Adler.



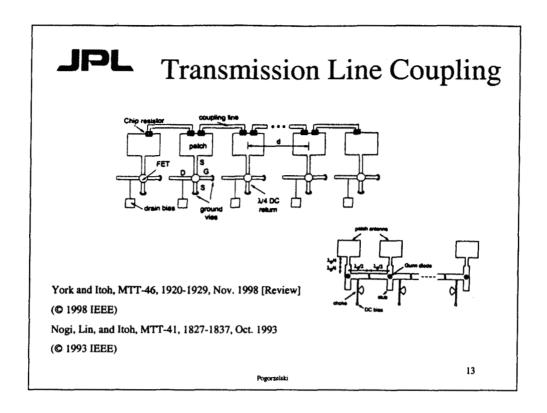
This is a ring array with unidirectional coupling designed to produce circular polarization. In steady state the 90 degree phase differences between the oscillators is exactly canceled by the 90 degree coupling phase resulting in phase locked oscillation at the free running frequency of the oscillators.



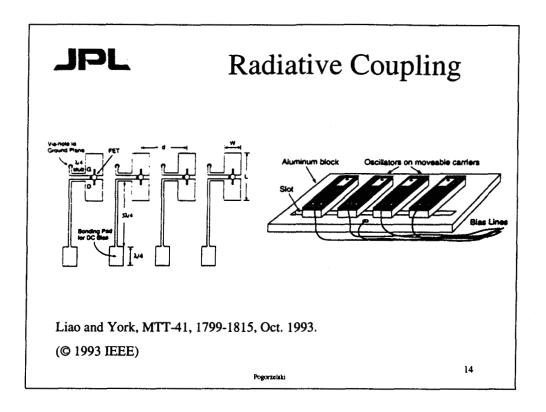
At low frequencies, one may consider using discrete component coupling networks instead of transmission lines. This can result in nearly zero coupling phase.



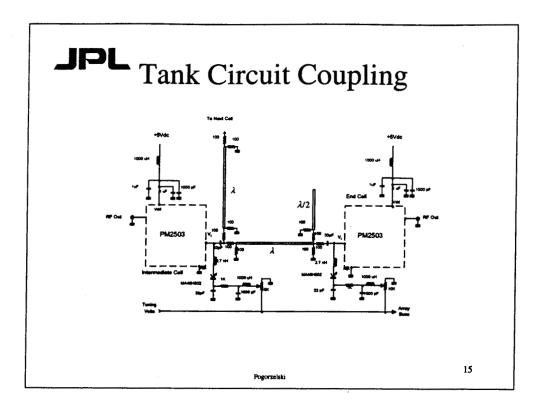
A similar low frequency scheme can be devised for two dimensional arrays as shown here. Stephan and Morgan have demonstrated steering via injection locking of the perimeter oscillators using such an arrangement.



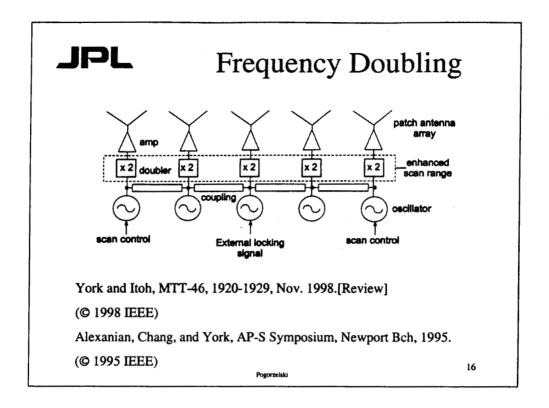
Initially, transmission line coupling was applied between the oscillator outputs by connection to the radiating elements themselves.



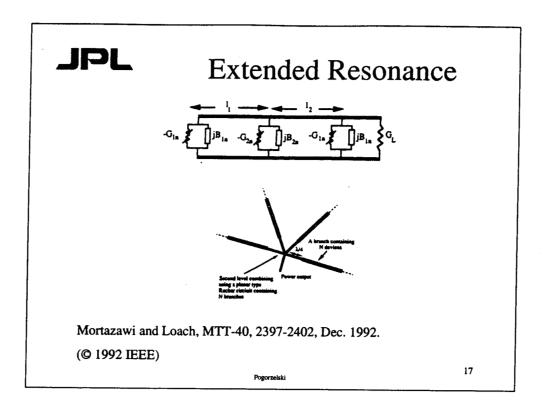
Oscillators can also be mutually injection locked via the radiating coupling between the radiating elements. This can be tricky because both the strength and phase of the coupling is dependent on the element spacing which, in turn, determines the radiation pattern.



More recently, transmission line coupling has been applied between the tank circuits of the oscillators and buffer amplifiers were employed to isolate the radiating aperture from the coupling network thus simplifying the design.



In order to maintain lock, any pair of locked oscillators must be no more than 90 degrees apart in phase. Thus, for an array with half wavelength spacing of the radiating elements, the maximum scan angle will be 30 degrees. If, however, the radiated frequency is twice the oscillator frequency, the maximum phase difference between locked oscillators becomes 180 degrees ideally permitting scanning to endfire.



Mortazawi and Loach propose a scheme in which the oscillators are not individually resonant but, instead, become resonant by virtue of their connection to neighboring oscillators in the array. This has been termed "extended resonance."



# Applications of Continuum Modeling

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The continuum model resulting in a partial differential equation governing the aperture phase of the array has been applied in analysis of one and two dimensional agile beam arrays using detuning and external injection for beam control. The following predicted transient and steady state behaviors have been obtained.

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### Beamsteering Dynamics

Equal and opposite detuning of the end oscillators; i.e.,

$$\Delta\omega_L = -\Delta\omega_R = \Delta\omega_T$$

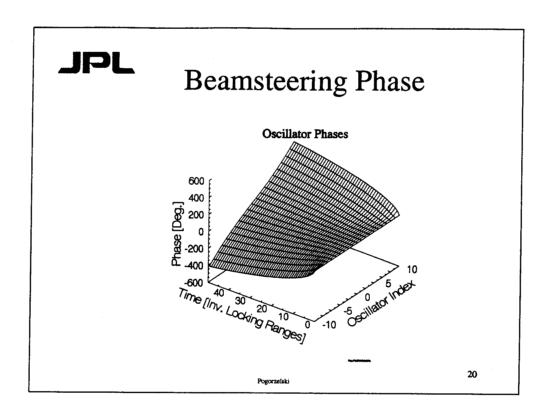
yields,

$$\phi(x,\tau) = \frac{\Delta\omega_T}{\Delta\omega_{lock}} \sum_{m=0}^{\infty} \frac{2\sin(b\sqrt{\sigma_m})\sin(x\sqrt{\sigma_m})}{(2a+1)\sigma_m} (1 - e^{-\sigma_m\tau})$$

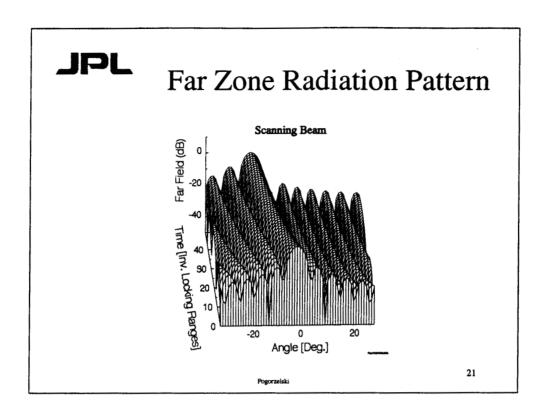
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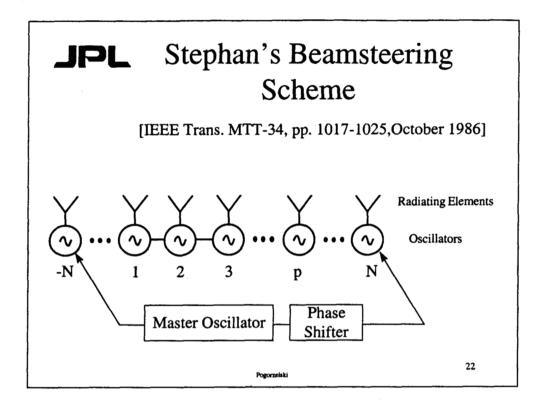
According to Liao, et.al. [IEEE Trans. MTT-41,pp. 1810-18115, Oct. 1993], beamsteering is accomplished by equal and opposite detuning of the end oscillators of the array. The solution for the phase distribution can be obtained from the solution for detuning one arbitrary oscillator (x=b) by superposition (subtraction) of two solutions, one for b=a and one for b=-a. The time domain result is as shown.



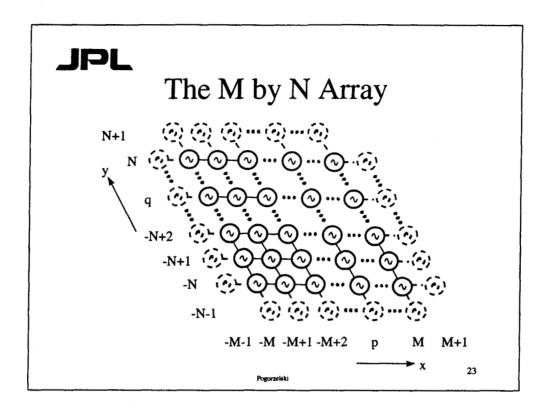
This is a graphical representation of the beamsteering phase solution just obtained.



This plot shows the dynamics of the far zone radiation pattern during beamsteering. It was obtained by computing the radiation pattern for each time value by integration over the aperture using the phase solution represented on the previous vugraph. Note that the beam integrity and sidelobe structure is maintained throughout the transient period.



This diagram shows the Stephan scheme for beam steering. The master oscillator provides injection signals to the two end oscillators while the phase shifter controls the relative phase of these signals. The result is a linear phase progression across the array.



This diagram schematically represents a (2M+1) by (2N+1) array of oscillators coupled to nearest neighbors. This is the array to be analyzed in the following. The oscillators shown in dashed lines are external sources which provide the properly phased injection signals to the perimeter oscillators of the array.

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### The Continuum Model

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - V(x, y)\phi - \frac{\partial \phi}{\partial \tau} = -\frac{\omega_{tune} - \omega_{ref}}{\Delta \omega_{lock}} - V(x, y)\phi_{inj}(x, y; \tau)$$

where,

$$\tau = \Delta \omega_{lock} \ t$$

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Thus, defining a continuous phi function and continuous variables x and y indexing the oscillators, we arrive at the partial differential equation for phi shown. As in the one dimensional case, V represents the distribution and strength of the injection signals with phase phi<sub>inj</sub>. Tau is time measured in inverse locking ranges.



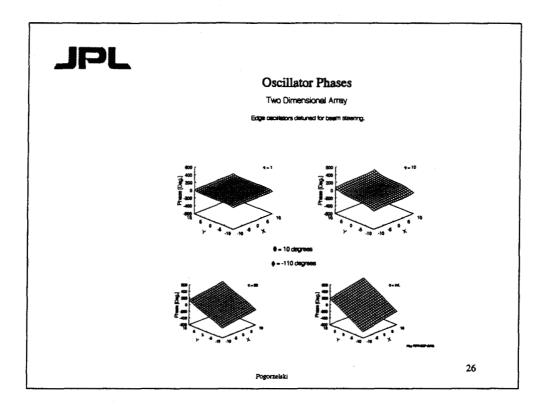
## A Numerical Example

- Consider a 21 by 21 element square array.
- Radiating elements:
  - Half wavelength spacing
  - Connected to each oscillator

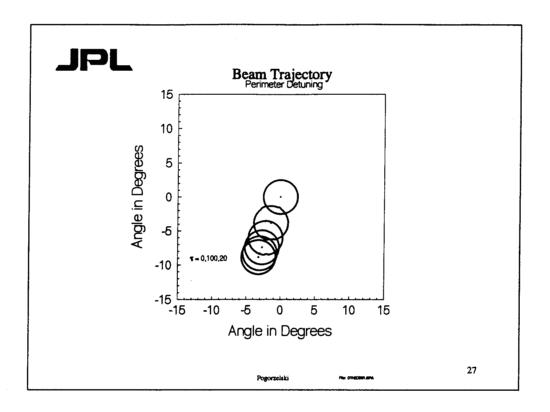
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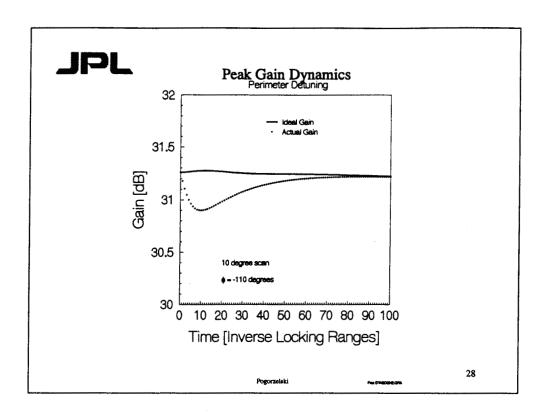
Consider a 21 by 21 element array with one radiating element connected to each oscillator. Let the radiating elements be spaced one half wavelength apart and let the external injection signals be applied to the perimeter oscillators per the preceding theory. The following vugraphs show a series of computed results concerning the aperture phase and far zone field of such an array.



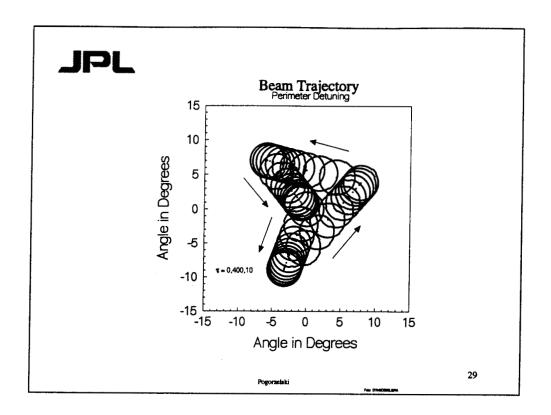
These graphs show the time evolution of the phase when detuning appropriate to beamsteering is applied. Note that the steering voltages are constant along each edge of the array.



This graph shows the beam peak (dots) and the three dB contour (closed curves) as a function of time during the beamsteering transient resulting when a step steering voltage designed to steer the beam thirty degrees off normal is applied at time zero.



During the transient period, the aperture phase is nonplanar. This results in a temporary reduction in gain due to phase aberration. This graph shows this gain reduction as a function of time compared with the projected aperture loss to be expected for each beam position. These curves were obtained by pattern integration.



This graph shows the result of four sets of steering voltages applied in rapid succession. Note that the aberration effects seem to be greater when steering from one off axis position to another than when steering to or from normal.

### JPL Summary of Key Results

- Inter-oscillator phase difference
  - Limited to 90 degrees.
  - Limit can be mitigated by:
    - · Reducing the element spacing.
    - · Adding oscillators between the radiating ones.
    - · Radiating at a harmonic of the coupling frequency.
- The response time of the array varies as the square of its size.
- The maximum step detuning of a single oscillator of a one dimensional array approaches two locking ranges for a large array.
- If all of the oscillators are externally injection locked or detuned the response time is that of a single oscillator.
- This linearized model substantiates the beam steering results of Stephan.
- The injection signal phase is limited to 90 degrees from injected oscillator phase unless applied gradually.
- The maximum step change in frequency of injection of one of the oscillators of a one dimensional array is limited to the locking range divided by the number of oscillators.

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In conclusion, the chart summarizes the key results obtained to date via continuum modeling concerning the dynamic behavior of one and two dimensional coupled oscillator arrays involving either detuning or external injection locking of the oscillators.



### An S-Band Experimental Array

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A seven element array operating at 2.53 GHz has been fabricated and tested. This array is described in the following vugraphs.



#### **PM 2503 MMIC**



PM2503 DATA SHEET

### RFIC OSCILLATOR 2000 to 3000 MHz Operation

#### **Features**

- 3 5 Volt Single Supply
- Output Power 14 dBm @ 5V
- Low Cost Surface Mountable
- Buffered AC Coupled Output



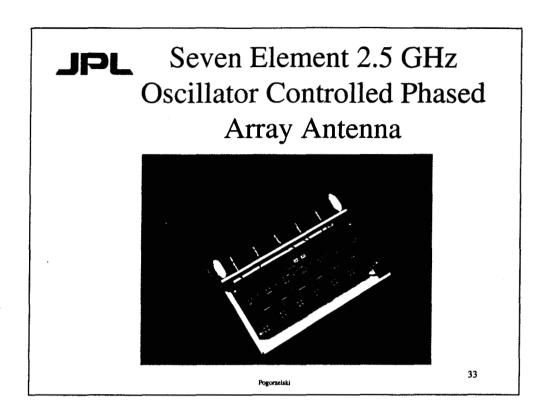
#### Description

The PM2503 is a GaAs negative resistance ("-R") RFIC that requires only a 3.0 - 5.0 Volt bias and a 40mA supply current. An external varactor diode and spiral inductor provide a low cost oscillator solution while providing 14 dBm output power in the 2000 to 3000 MHz frequency range. The PM2503 contains a fundamental oscillator, integrated matching network, buffer amplifier and all bias networks. Potential applications include communication systems, transmitters, receivers, and other systems requiring a small easy to use source.

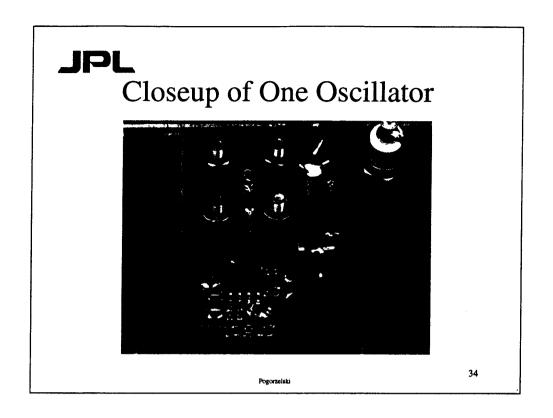
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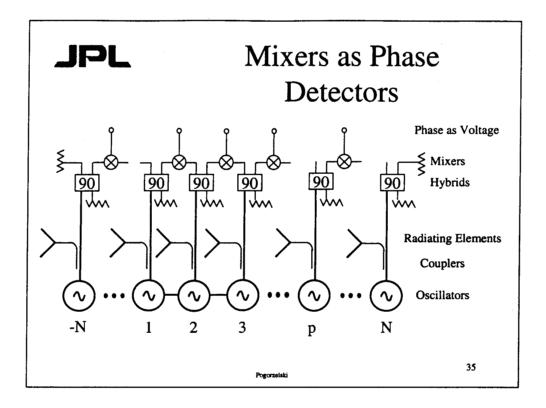
With the consultation of the UCSB Group (R. A. York and P. F. Maccarini) the PM2503 RFIC was selected for use in constructing a laboratory model array at S-Band (2.5 GHz).



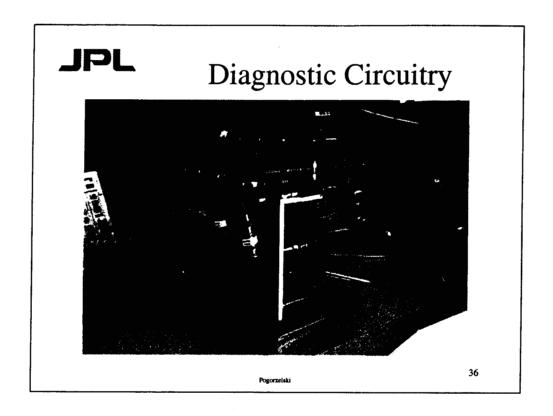
The shows the oscillator array mated with the seven element patch array. The shorting bars are clearly visible and the excess transmission line has been eliminated by cutting the line at the bar. There, of course, remains a discontinuity due to the difference in impedance between the line and the bar.



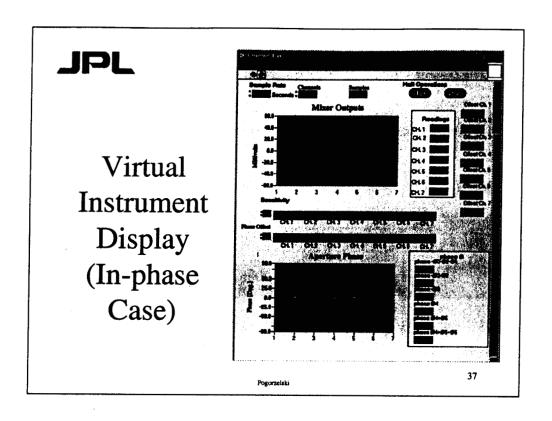
This is a closeup of one of the oscillators showing the tuning potentiometer and binding post. Also visible are the inductors used to isolate the power supply from the rf.



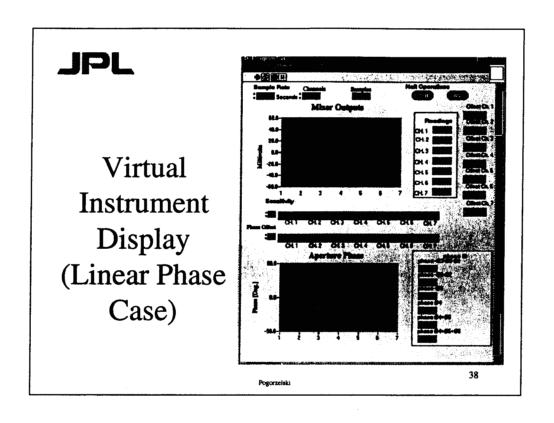
The experimental setup for verifying the theoretically predicted array behavior makes use of mixers as phase detectors as shown here. The 90 degree hybrids are used to make the mixer outputs zero when the corresponding two oscillators are in phase. (Without the hybrids, the output would be zero for a 90 degree phase difference.) Ten dB couplers are used to derive the signals to be sent to the radiating elements. While one might expect that the mixer signals would be derived in this manner instead, the present arrangement provides adequate signal for driving the mixers while retaining the ability to measure radiation patterns since the receiver is more sensitive than the mixers.



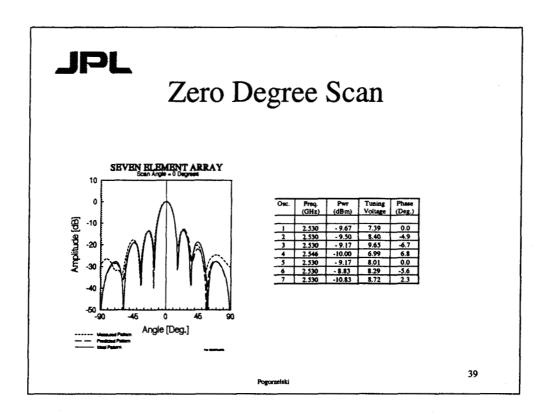
This is a photograph of the array with the added diagnostic circuitry.



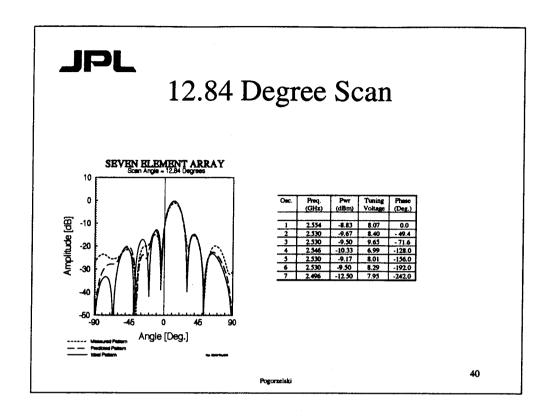
The mixer outputs are read by a "Virtual Instrument" implemented in LabView. The display is shown above for tuning which yields a uniform aperture phase distribution.



If the end oscillators are detuned oppositely, a linear phase distribution results as shown here.



Three patterns are shown here for zero degree scan. The solid line is the ideal pattern of the seven element array. The dashed line is the pattern predicted from the measured amplitude and phase of the patch excitations shown in the chart. Finally, the dotted line is the measured pattern under oscillator excitation of the patch array.



This final scan case shows the beam nearly 13 degrees off boresight. Beyond this point, the amplitude of oscillator number 7 drops to too low a level to participate in the interaction and loses lock.

# Summary of the Experiment

- A coupled oscillator controlled phased array has been designed, fabricated, and evaluated.
  - PM2503 oscillators MMICs
  - Seven patch elements
  - Linear (one dimensional) array
  - Transmit only
- The theoretical predictions of coupling phase effects were verified.
- The beamsteering range was limited by amplitude variation with tuning.

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While this array is transmit only, a receive array can be designed using the same principles. One would use the oscillators of the array to provide local oscillator signals to be mixed with the signals from each receive element in the aperture. These local oscillator signals would carry the phase necessary to render the received signals cophasal for incidence from a given direction. The direction is, of course, determined by detuning the end oscillators of the array just as in the transmit case.



#### Modulation

- To impose information on the radiated signal, the array must be modulated.
- Frequency modulation is the most natural scheme.
- The question is, how?

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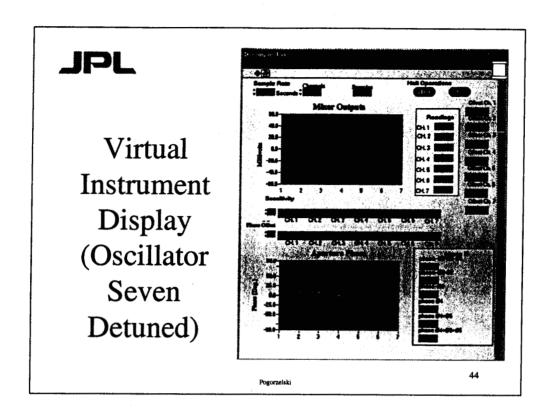
## Frequency Modulation

- Appear to be three options for frequency modulation.
  - Modulate one oscillator.
  - Modulate more than one oscillator.
  - Modulate all of the oscillators.
- The first two options can be immediately discarded because the steady state phase distribution would be parabolic.

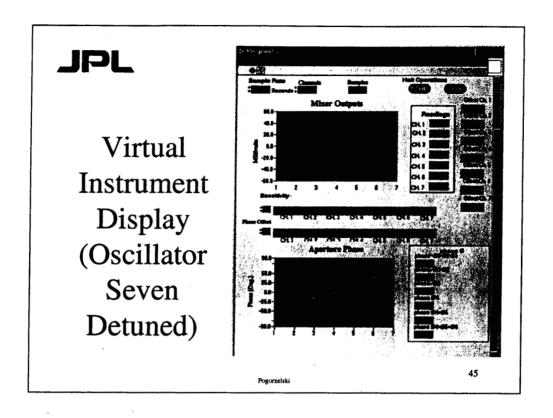
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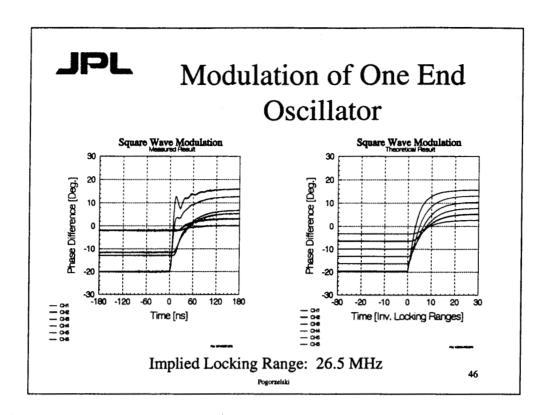
It will be shown in the following that to effectively place information on the radiated beam it is necessary to modulated all of the oscillators in the array simultaneously.



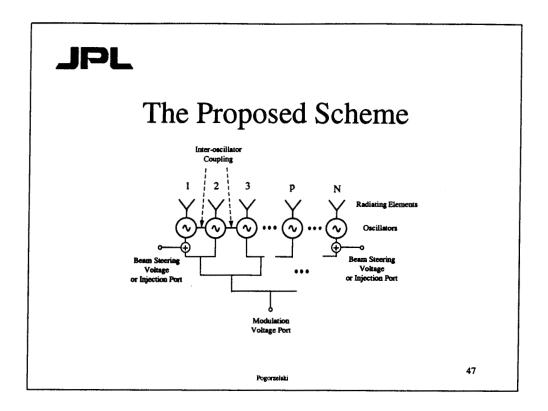
If only oscillator seven is detuned, a parabolic distribution results.



Detuning oscillator seven in the opposite direction also produces a parabolic distribution. Modulation of oscillator seven with a square wave switches between this and the preceding distribution and permits observation of the transient behavior of the array.



Using the system illustrated in the preceding vugraphs, the above results on the left were obtained by modulating oscillator 7 with a .2 volt peak to peak square wave injected into the oscillator tank circuit between the varactor and the resonating inductor through a very large (0.1 microFarad) capacitor. The mixers were calibrated to permit conversion of the measured output voltage to degrees of phase difference between adjacent oscillators. The corresponding theoretical prediction is shown on the right. Comparison of the plots permits confirmation of the locking range of the oscillators.



We propose to implement modulation of all the oscillators using the network shown. While resembling a corporate feed network, the line lengths here are not critical, as they would be at rf, because the network operates at baseband.



- Modulation of one oscillator is ineffective.
  - Steady state phase distribution is parabolic.
  - Average locking range can be inferred from transients.
  - Theoretical predictions experimentally verified.
- All oscillators must be modulated.
  - Steady state phase distribution is linear.
  - Transient response is that of one oscillator.

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From the presented results, we can conclude that the theoretical predictions are born out in the measurements and that both imply that effective modulation can only be achieved by simultaneously modulating all of the oscillators in the array.



## Current Work (at JPL)

- L-Band Receive Array
  - 15 oscillators
  - 9 Radiating Elements
- S-Band Two Dimensional Array
  - 25(?) Oscillators
  - 25(?) Radiating Elements

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We are currently funded by BMDO to build an L-band receive array based on coupled oscillator principles. It is intended that the scan range will exceed 30 degrees because radiating elements will be connected only to every other oscillator.

We are also funded by NASA to begin work on two dimensional arrays. This work will be done at S-band due to ready availability of components. The size of the arrays has not yet been determined.



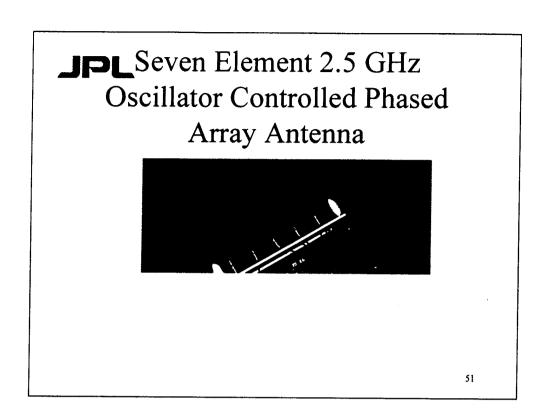
## Future Work (at JPL)

- Larger two dimensional arrays.
- Higher frequency arrays.
  - Ka-Band
  - Collaboration with Clemson University
- Transmit/receive arrays.
  - Shared aperture
  - Shared oscillator array

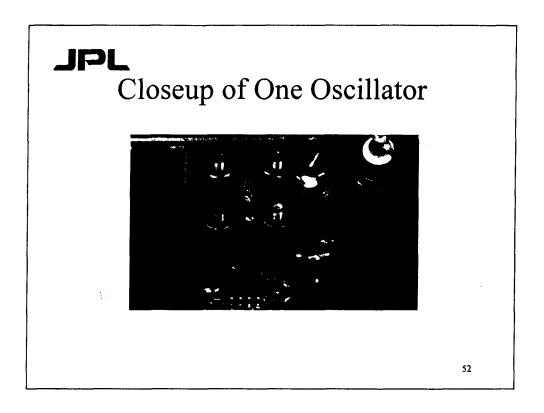
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Our plan includes further work in two dimensions, development of higher frequency arrays in collaboration with Pearson's group at Clemson, and development of arrays which both transmit and receive.



The shows the oscillator array mated with the seven element patch array. The shorting bars are clearly visible and the excess transmission line has been eliminated by cutting the line at the bar. There, of course, remains a discontinuity due to the difference in impedance between the line and the bar.



This is a closeup of one of the oscillators showing the tuning potentiometer and binding post. Also visible are the inductors used to isolate the power supply from the rf.

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## **Coupling Phase Effects**

$$\begin{split} \frac{d\theta_{i}}{dt} &= \omega_{\text{nume},i} - \sum_{\substack{j=i-1\\j \neq i}}^{i+1} \Delta \omega_{\text{lock},ij} \sin(\Phi_{ij} + \theta_{i} - \theta_{j}) \\ \frac{d\theta_{i}}{dt} &= \omega_{\text{nume},i} - \sum_{\substack{j=i-1\\j \neq i}}^{i+1} \Delta \omega_{\text{lock},ij} \left[ \sin(\Phi_{ij}) \cos(\theta_{i} - \theta_{j}) + \cos(\Phi_{ij}) \sin(\theta_{i} - \theta_{j}) \right] \\ \frac{d\theta_{i}}{dt} &= \omega_{\text{nume},i} - \sum_{\substack{j=i-1\\j \neq i}}^{i+1} \Delta \omega_{\text{lock},ij} \left[ \sin(\Phi_{ij}) + \cos(\Phi_{ij}) (\theta_{i} - \theta_{j}) \right] \\ \frac{d\theta_{i}}{dt} &= \omega_{\text{nume},i} - \sum_{\substack{j=i-1\\j \neq i}}^{i+1} \Delta \omega_{\text{lock}} \left[ \sin(\Phi) + \cos(\Phi) (\theta_{i} - \theta_{j}) \right] \end{split}$$

The theory predicts some interesting relations between the coupling phase on the one hand and the array locking range and ensemble frequency on the other.

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Beginning with the system of nonlinear first order differential equations presented by York one can use trigonometric identities to identify these effects.

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### Coupling Phase Effects (Cont.)

$$\theta_i = \omega_{ref}t + \phi_i$$

$$\frac{d\phi_i}{dt} = \omega_{tune,i} - \langle \omega_{tune,i} \rangle - \sum_{\substack{j=i-1\\j \neq i}}^{i+1} \Delta \omega_{lock} \left[ \sin(\Phi) + \cos(\Phi)(\phi_i - \phi_j) \right]$$

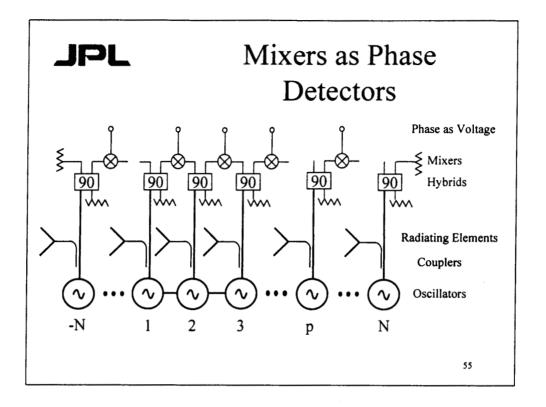
$$\frac{d\phi_{i}}{dt} = \left(\omega_{tune,i} - \langle \widetilde{\omega}_{tune,i} \rangle\right) - \sum_{\substack{j=i-1\\j\neq i}}^{i+1} \Delta \widetilde{\omega}_{lock} (\phi_{i} - \phi_{j})$$

$$<\widetilde{\omega}_{lune,i}>=<\omega_{lune,i}>+\Delta\omega_{lock}\sin(\Phi)$$
  $\Delta\widetilde{\omega}_{lock}=\Delta\omega_{lock}\cos(\Phi)$ 

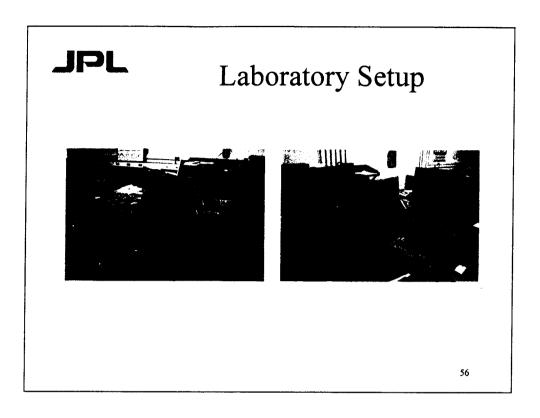
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Defining the phase relative to a reference frequency, one can group terms and factors so as to define an effective ensemble frequency and effective locking range as shown. Note that the locking range is maximum when the coupling phase is a multiple of pi in which case the ensemble frequency is equal to the average of the free running frequencies.

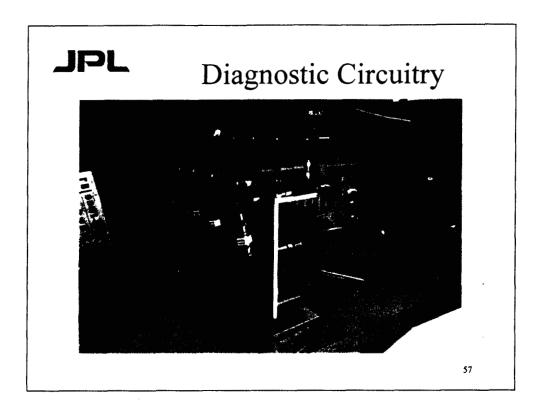
These effects were easily observed during the shorting bar adjustment.



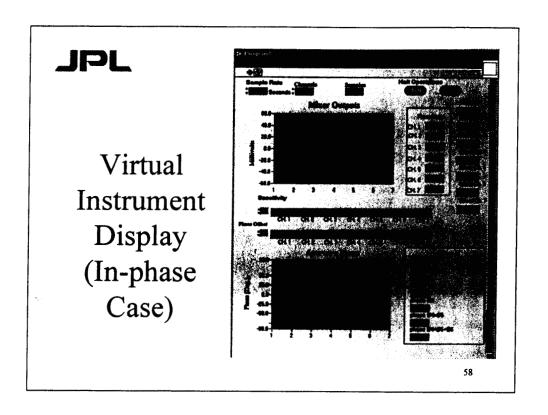
The experimental setup for verifying the theoretically predicted array behavior makes use of mixers as phase detectors as shown here. The 90 degree hybrids are used to make the mixer outputs zero when the corresponding two oscillators are in phase. (Without the hybrids, the output would be zero for a 90 degree phase difference.) Ten dB couplers are used to derive the signals to be sent to the radiating elements. While one might expect that the mixer signals would be derived in this manner instead, the present arrangement provides adequate signal for driving the mixers while retaining the ability to measure radiation patterns since the receiver is more sensitive than the mixers.



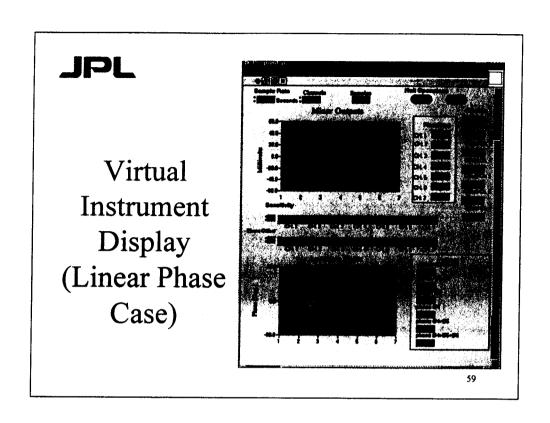
This is a photograph of the laboratory equipment used to diagnose the array behavior.



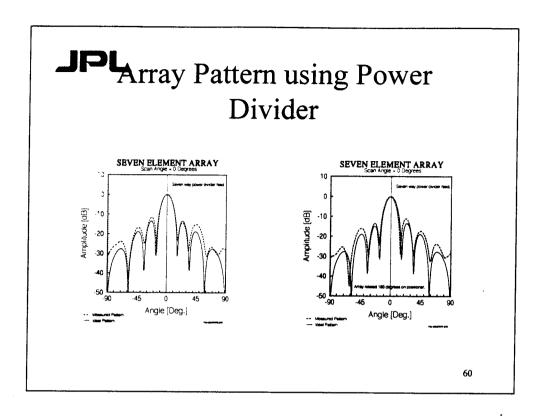
This is a photograph of the array with the added diagnostic circuitry.



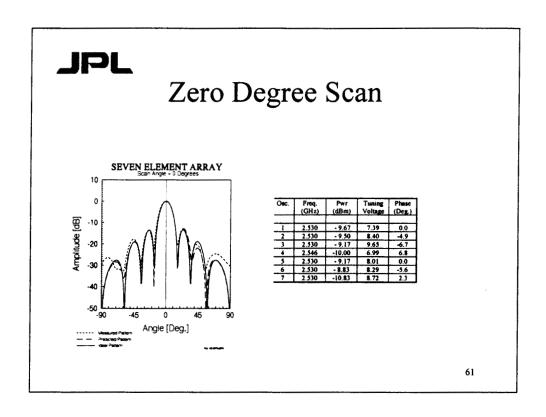
The mixer outputs are read by a "Virtual Instrument" implemented in LabView. The display is shown above for tuning which yields a uniform aperture phase distribution.



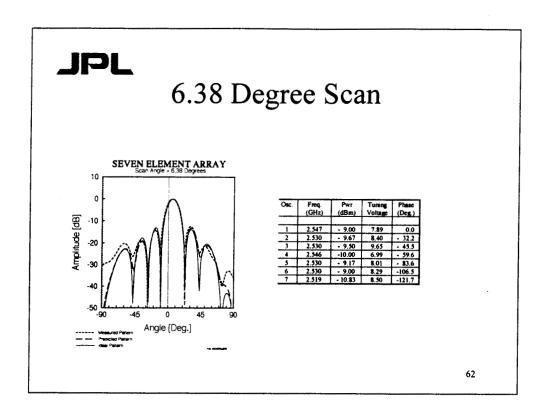
If the end oscillators are detuned oppositely, a linear phase distribution results as shown here.



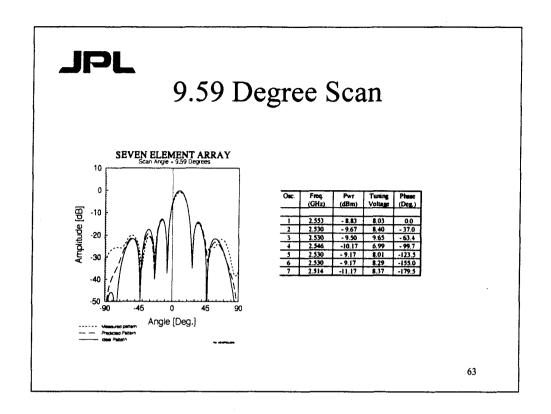
As a test of the radiating aperture, uncontaminated with coupled oscillator array effects, a pattern was measured with the aperture excited via an eight way power divider with one port terminated and the other seven connected to the patch elements. A second pattern was measured with the array rotated 180 degrees about boresight to discern possible range effects. These graphs compare the measured patterns with the theoretical ideal. The agreement is quite satisfactory and range artifacts are only evident below -25 dB.



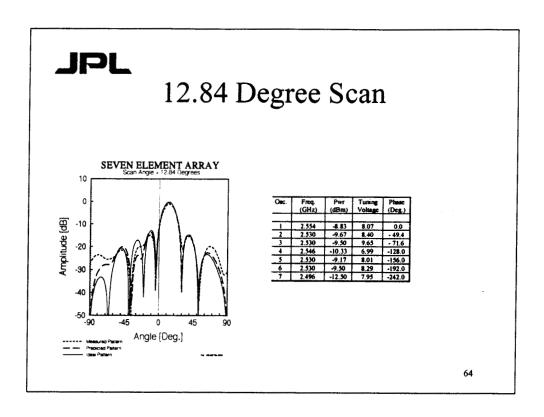
Three patterns are shown here for zero degree scan. The solid line is the ideal pattern of the seven element array. The dashed line is the pattern predicted from the measured amplitude and phase of the patch excitations shown in the chart. Finally, the dotted line is the measured pattern under oscillator excitation of the patch array.



These curves are similar to those in the last vugraph but, this time, for a scanned beam. As can be seen from the chart this scan was achieved by changing the tuning voltage on the end oscillators.



Here the tuning voltages are further adjusted to produce a larger scan angle.



This final scan case shows the beam nearly 13 degrees off boresight. Beyond this point, the amplitude of oscillator number 7 drops to too low a level to participate in the interaction and loses lock.

# JPL Summary of the Experiment

- A coupled oscillator controlled phased array has been designed, fabricated, and evaluated.
  - PM2503 oscillators MMICs
  - Seven patch elements
  - Linear (one dimensional) array
  - Transmit only
- The theoretical predictions of coupling phase effects were verified.
- The beamsteering range was limited by amplitude variation with tuning.

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While this array is transmit only, a receive array can be designed using the same principles. One would use the oscillators of the array to provide local oscillator signals to be mixed with the signals from each receive element in the aperture. These local oscillator signals would carry the phase necessary to render the received signals cophasal for incidence from a given direction. The direction is, of course, determined by detuning the end oscillators of the array just as in the transmit case.



#### Modulation

- To impose information on the radiated signal, the array must be modulated.
- Frequency modulation is the most natural scheme.
- The question is, how?

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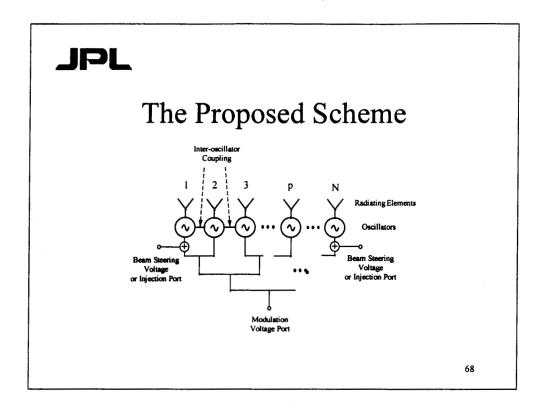


## Frequency Modulation

- Appear to be three options for frequency modulation.
  - Modulate one oscillator.
  - Modulate more than one oscillator.
  - Modulate all of the oscillators.
- The first two options can be immediately discarded because the steady state phase distribution would be parabolic.

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It will be shown in the following that to effectively place information on the radiated beam it is necessary to modulated all of the oscillators in the array simultaneously.



We propose to implement modulation of all the oscillators using the network shown. While resembling a corporate feed network, the line lengths here are not critical, as they would be at rf, because the network operates at baseband.

#### JPL

### Consider a Square Wave

The source term becomes,

$$h_{\text{mod}} = \frac{\pi}{2}u(\tau) + \sum_{n=1}^{\infty} \pi (-1)^n u(\tau - n\frac{T}{2})$$

The Laplace transform is,

$$H(s) = \frac{1}{s} \tanh\left(\frac{s}{4}\right)$$

and we wish to solve,

$$\frac{\partial^2 F}{\partial x^2} - sF = -\frac{1}{s} \tanh\left(\frac{s}{4}\right)$$

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The behavior of the proposed array under square wave modulation can be ascertained theoretically using the diffusion equation presented earlier. The source term is  $h_{mod}$ . Solution is effected via the Laplace transform. This source waveform has a known Laplace transform shown here and the resulting transformed equation is given at the bottom of the vugraph.

JPL

# Square Wave Continued

A particular integral is,

$$F_p(x,s) = \frac{1}{s^2} \tanh\left(\frac{s}{4}\right)$$

Adding two complementary functions gives,

$$F(x,s) = \frac{1}{s^2} \tanh\left(\frac{s}{4}\right) + Ae^{\sqrt{s}x} + Be^{-\sqrt{s}x}$$

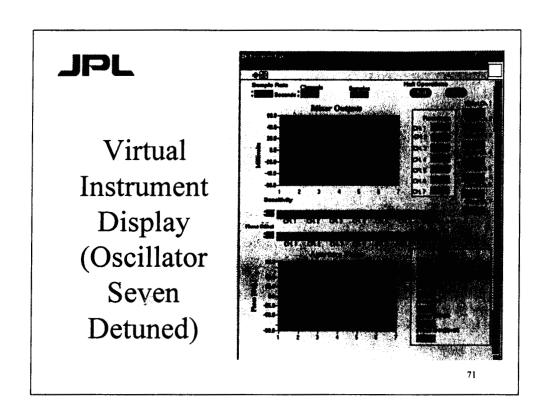
Boundary conditions require that A and B be zero so the inverse transform becomes,

$$\phi(\tau) = \tau \frac{\pi}{2} u(\tau) + \sum_{n=1}^{\infty} \pi (-1)^n (\tau - n \frac{T}{2}) u(\tau - n \frac{T}{2})$$

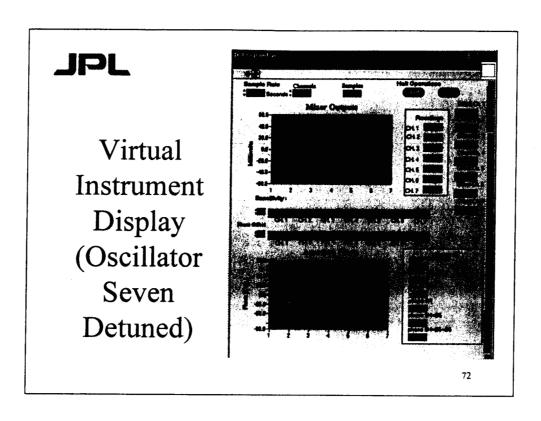
That is, the array integrates the modulation signal.

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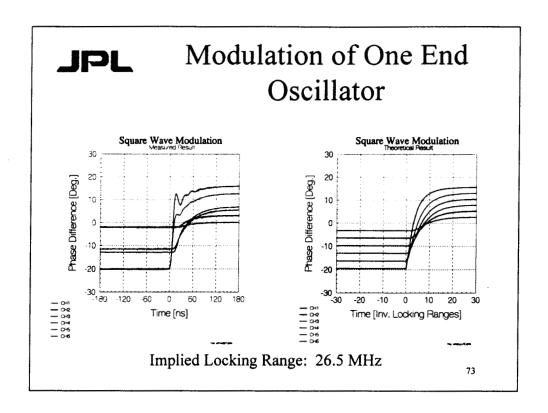
Proceeding with the solution, we add to a particular integral the appropriate amounts of homogeneous solutions to satisfy the boundary conditions (Neumann conditions) at the array ends. The result shown here indicates that the array transmits a frequency modulated signal. For phase modulation the array basically integrates the modulation signal. Thus, we propose that in this case the information signal be first differentiated and then applied to the array modulation network resulting in transmission of an appropriately phase modulated signal.



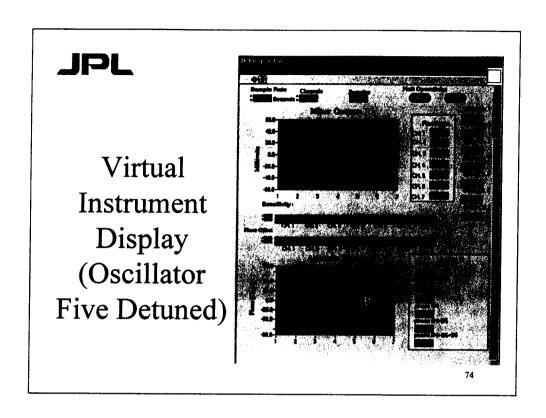
If only oscillator seven is detuned, a parabolic distribution results.



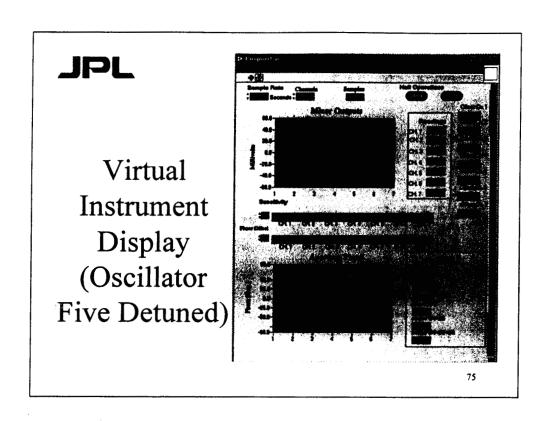
Detuning oscillator seven in the opposite direction also produces a parabolic distribution. Modulation of oscillator seven with a square wave switches between this and the preceding distribution and permits observation of the transient behavior of the array.



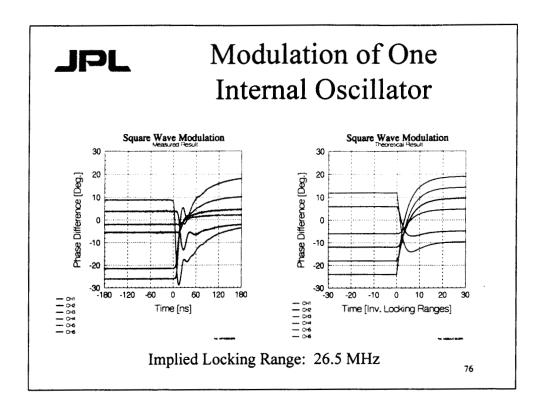
Using the system illustrated in the preceding vugraphs, the above results on the left were obtained by modulating oscillator 7 with a .2 volt peak to peak square wave injected into the oscillator tank circuit between the varactor and the resonating inductor through a very large (0.1 microFarad) capacitor. The mixers were calibrated to permit conversion of the measured output voltage to degrees of phase difference between adjacent oscillators. The corresponding theoretical prediction is shown on the right. Comparison of the plots permits confirmation of the locking range of the oscillators.



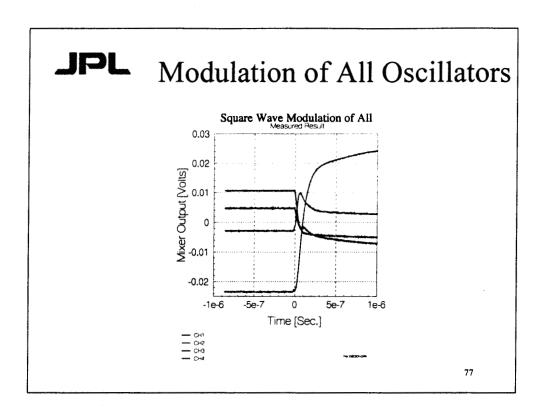
If only oscillator five is detuned, a dual parabolic distribution results.



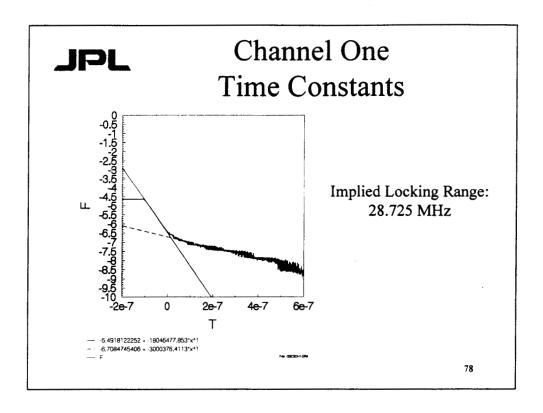
Detuning oscillator five in the opposite direction also produces a dual parabolic distribution. Modulation of oscillator five with a square wave switches between this and the preceding distribution and permits observation of the transient behavior of the array.



Using the system illustrated in the preceding vugraphs, the above results on the left were obtained by modulating oscillator 5 with a .2 volt peak to peak square wave injected into the oscillator tank circuit between the varactor and the resonating inductor through a very large (0.1 microFarad) capacitor. The corresponding theoretical prediction is shown on the right. Comparison of the plots again confirms the locking range of the oscillators.



If all of the oscillators are modulated simultaneously one should theoretically observe zero phase differences. Thus, in order to see the transients, we detuned the oscillators to produce observable phase differences and then applied the modulation to all of them. This permits the determination of the time constants from which the locking range can again be computed. Note that the short time constants are the ones relevant to the locking range computation. The long time constants are related to the large bypassing components used in the tuning power supply circuit.



Selecting the channel 1 signal from the preceding graph, we fit the natural logarithm of the function to a set of straight lines whose slopes give the time constants involved. The long time constant is related to the large bypassing components used on the tuning supply. The shorter time constant implies a locking range of 28.7 MHz which is quite constant (within the experimental error) with the 26.5 MHz obtained previously and with the result of direct measurement of the locking range.

# JPL Remarks on Modulation

- Modulation of one oscillator is ineffective.
  - Steady state phase distribution is parabolic.
  - Average locking range can be inferred from transients.
  - Theoretical predictions experimentally verified.
- All oscillators must be modulated.
  - Steady state phase distribution is linear.
  - Transient response is that of one oscillator.

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From the presented results, we can conclude that the theoretical predictions are born out in the measurements and that both imply that effective modulation can only be achieved by simultaneously modulating all of the oscillators in the array.

#### JPL

#### Current Work (at JPL)

- L-Band Receive Array
  - 15 oscillators
  - 9 Radiating Elements
- S-Band Two Dimensional Array
  - 25(?) Oscillators
  - 25(?) Radiating Elements

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We are currently funded by BMDO to build an L-band receive array based on coupled oscillator principles. It is intended that the scan range will exceed 30 degrees because radiating elements will be connected only to every other oscillator.

We are also funded by NASA to begin work on two dimensional arrays. This work will be done at S-band due to ready availability of components. The size of the arrays has not yet been determined.



## Future Work (at JPL)

- Larger two dimensional arrays.
- Higher frequency arrays.
  - Ka-Band
  - Collaboration with Clemson University
- Transmit/receive arrays.
  - Shared aperture
  - Shared oscillator array

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Our plan includes further work in two dimensions, development of higher frequency arrays in collaboration with Pearson's group at Clemson, and development of arrays which both transmit and receive.